

Basic Properties of Open-Channel Flows

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1. Geometric Elements

- flow depth (h): the vertical distance of the lowest point of a channel section from the free surface
- stage: the elevation of the free surface above a datum
- top width (T): the width of channel section at the free surface
- water area (A): the cross sectional area of the flow normal to the flow of direction
- wetted perimeter (P): the length of the line of intersection of the channel wetted surface with a cross sectional plane normal to the flow direction. The wetted perimeter is related to the shear stress acting in the opposite direction of the flow

- hydraulic radius (R_h): the ratio of the water area to its wetted perimeter

$$R_h = A / P$$

- hydraulic depth (D): the ratio of the water area to the top width

$$D = A / T$$

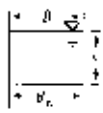



- section factor for critical flow computation (Z):

$$Z = A\sqrt{D}$$

- prismatic channel: a channel with unvarying cross-section and constant slope

Table 1. Geometric elements of channel sections (p.21, Chow)

TABLE 5.6.1
Geometric Functions for channel elements

Section:	Rectang.	Trapezoid	Triangl.	Circle
				
Area A	B_y	$(B_b + z)y$	$\frac{1}{2}y^2$	$\frac{1}{8}(\theta - \sin \theta)d^2$
Wetted perimeter P	$B_b + 2y$	$B_b + 2y\sqrt{1+z^2}$	$2y\sqrt{1+z^2}$	$\frac{1}{2}d\theta$
Hydraulic radius R	$\frac{B_y}{B_b + 2y}$	$\frac{(B_b + z)y}{B_b + 2y\sqrt{1+z^2}}$	$\frac{y}{2\sqrt{1+z^2}}$	$\frac{1}{4}\left[1 - \frac{\sin \theta}{\theta}\right]d$
Top width B	B_b	$B_b + 2zy$	$2zy$	$\left[\cos\left(\frac{\theta}{2}\right)\right]d$
				$2\sqrt{y(d-y)}$
$\frac{dR}{dy}$	$\frac{1}{A} \frac{dA}{dy}$	$\frac{5B_b + 6y}{3y(B_b + 2y)}$	$\frac{B_b + 2zy(5B_b + 6y\sqrt{1+z^2}) + 4zy^2\sqrt{1+z^2}}{3y(B_b + 2y\sqrt{1+z^2})}$	$\frac{4(2\sin \theta + 3\theta - 5\theta \cos \theta)}{3d\theta^2 - \sin \theta} \sin^2(\theta/2)$
				where $\theta = 2 \cos^{-1} \left[\frac{2y}{d} \right]$

Source: Chow, V. T., *Open-Channel Hydraulics*, McGraw-Hill, New York, 1959, Table 4.1, p. 21 (with additions)

2. Classification of Flows

2.1 Types of Flows

(1) boundary: closed conduit vs. open-channel flows

The driving forces of these two flows are different. The presence of the free surface makes the flow structure quite complicated. In general, the sewer flow is an open-channel flow if it has a free surface (or if it is not full). However, in a sewer flow, the effect of air cannot be ignored if more than 80 % of the total pipe area is occupied by the water.

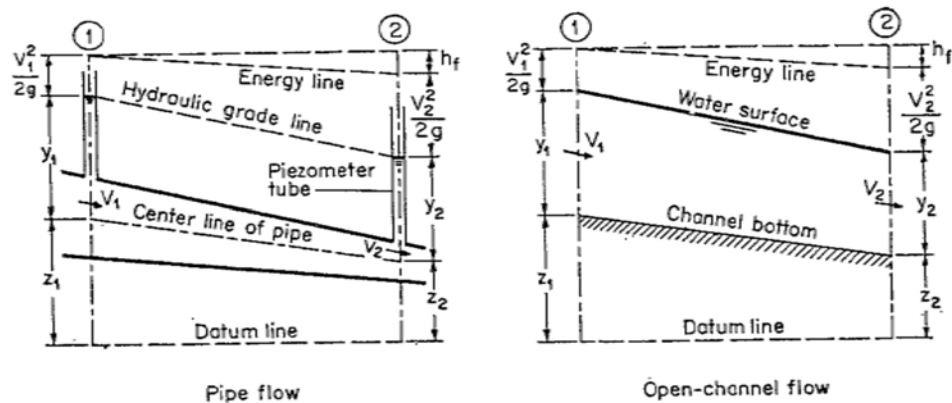


Figure 1. Comparison between pipe flow and open-channel flow (Chow, 1959)

(2) time: steady vs. unsteady flows

Flow oscillations are not related with flow unsteadiness but with turbulence.

(3) space: uniform vs. non-uniform flows and gradually varied vs. rapidly varied flows

Depending on the level of change in water surface, we divide the non-uniform flow into gradually varied flow and rapidly varied flow.

(4) viscosity: laminar vs. turbulent flows

The Reynolds number is the indicative of the dominance of the fluid viscosity in the flow.

(5) gravity: subcritical vs. supercritical flows

The Froude number is the indicative of the dominance of the gravity in the flow.

(6) fluids: single phase vs. multi-phase flows and air (gas)-water flow and fluid layers

Open-channel flow is an air-water multi-phase flow, and salt water intrusion is fluid layers.

2.2 Unsteadiness and Uniformity

Based on the flow unsteadiness and uniformity, we can classify the open-channel flow into four categories. They are

- 1) steady uniform flow
- 2) steady non-uniform flow
- 3) unsteady uniform flow
- 4) unsteady non-uniform flow

Since the unsteady uniform flow is practically impossible, we have only three. For convenience's sake, the three flows can be referred to as

- 1) uniform flow

2) non-uniform flow

4) unsteady flow

So non-uniform flows are steady, and unsteady flows are non-uniform.

3. Uniform Flow

The uniform flow is the flow whose flow depth (or water area), velocity, and discharge are constant with distance. Thus, the energy line, water surface, and channel bottom are all parallel. Note that uniform flow is considered steady only, since unsteady uniform flow is practically non-existent.

In the uniform flow, all streamlines are parallel. For the laminar flow, the streamline, pathline, streakline are the same. For the turbulent flow, streamlines averaged over time and space are parallel.

4. Varied Flow

The flow is rapidly varied if the depth changes abruptly over a comparatively short distance; otherwise it is gradually varied. Examples of rapidly varied flows are hydraulic jump and hydraulic drop.

A question arises that how gradual the flow should change in the gradually varied flow. This can be answered by investigating the definition of GVF, where two assumptions are involved in the definition. First, the flow is almost steady - hydraulic properties of the flow remain

constant for the time interval. Second, the streamlines are practically parallel - the hydrostatic pressure prevails over the channel reach. Note that the unsteady flows are also divided into unsteady GVF (flood wave) and unsteady RVF (moving bore).

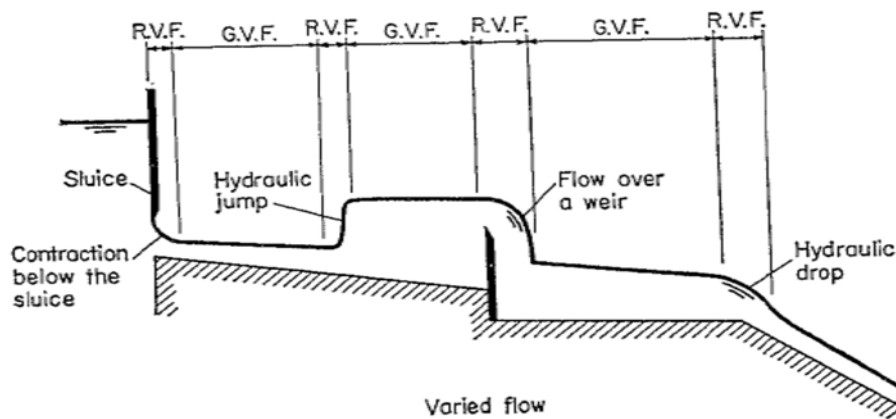


Figure 2. Non-uniform flows (Chow, 1958)

5. Critical Reynolds Number

According to Reynolds' 1883 experiments, pipe flow is thought to be critical when its Reynolds number reaches 2000. Here, the Reynolds number is defined by

$$Re = \frac{VD}{\nu} \quad (1)$$

where V = average velocity, D = pipe diameter, and ν = kinematic viscosity of fluids. However, for open-channel flows, we cannot use this criterion because no diameter is available for the open-channel flow.

We can obtain the critical value of the Reynolds number for the open-channel flow by using the concept of characteristic length. For the pipe flow, the diameter is used for the characteristic length, which is a good choice. However, for the open-channel flow, we do not have a geometric element such as the diameter for the pipe flow due to the presence of the water surface.

Considering the water surface, the following hydraulic radius is an excellent choice for the characteristic length for the open-channel flow:

$$R_h = \frac{A}{P} \quad (2)$$

where A = water area and P = wetted perimeter. Since $R_h = D/4$, the critical Reynolds becomes 500 if the diameter is replaced with the hydraulic radius. Now, this value can be used for the critical Reynolds number for the open-channel flow.

6. Multiphase Flow

The open-channel flow is a multiphase flow in a strict sense. It is an air-water multiphase flow. However, air is not important in the flow structure because its density is about 770 times lighter than water ($\rho_{water} = 1 \text{ g/cm}^3$ and $\rho_{air} = 0.0013 \text{ g/cm}^3$). Therefore open-channel flows can be regarded as a single phase flow. In a sewer flow, the effect of air cannot be ignored if more than 80 % of the total pipe area is occupied by water.

An example of water layer flows is a stream intrusion into reservoir. When the stream flow enters into the reservoir, it is not mixed readily with the surrounding or ambient water, thus forming the stratification. In the spring time, the temperature of the incoming water from the stream is relatively higher than that stored in the reservoir, and the overflow occurs.

However, in the summer time, the temperature of the incoming water is lower than that stored in the reservoir which has been heated during the day time. Thus, the underflow takes place.

In the deep reservoir, if the underflow finds the water layer whose density is the same when it propagates along the bottom of the reservoir, then it propagates horizontally. It is called the interflow.

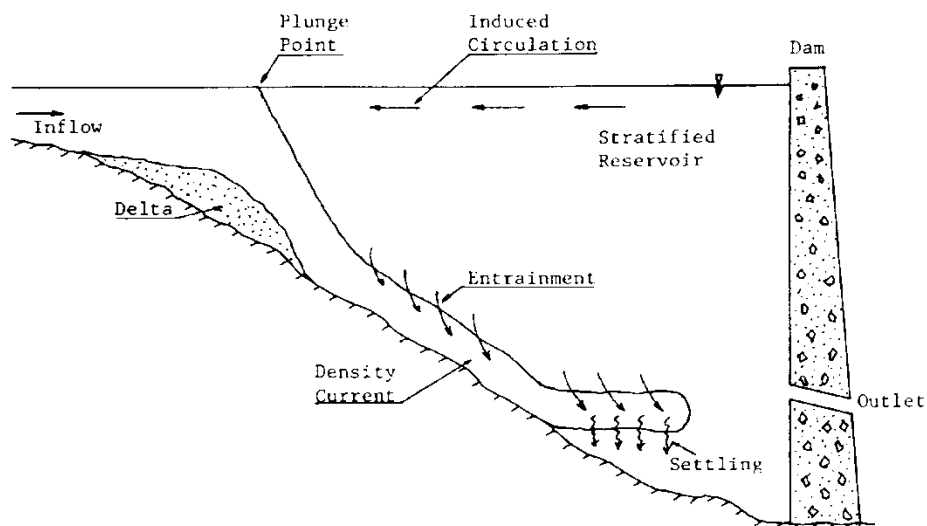


Figure 3. Stream intrusion into a reservoir

7. Velocity Distribution in a Channel Section

The velocity is not uniform in the channel cross section due to the presence of the free surface and sidewalls. Rather the velocity distribution looks chaotic even for flows at a very

low Re . Unless the channel is wide, the velocity maximum occurs below the water surface. This is called the velocity dip. This phenomenon is related with the secondary currents.

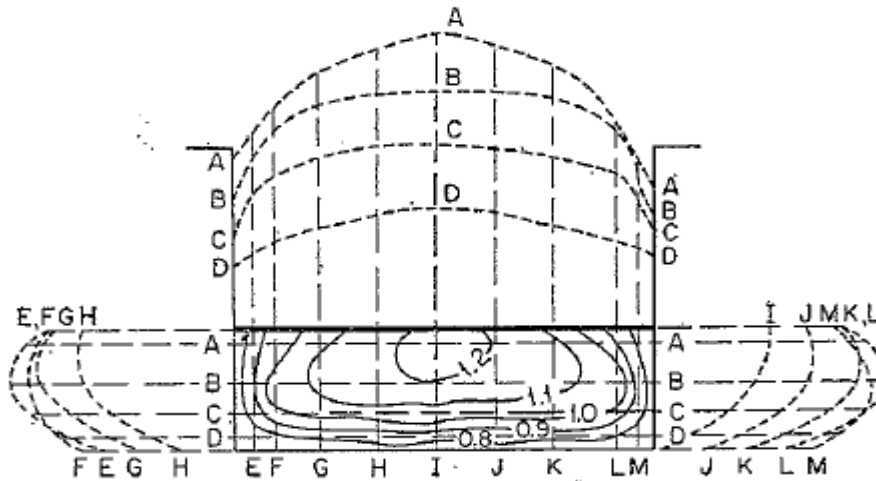


Figure 4. Isovels in a rectangular channel (Chow, 1959)

Contrary to the general belief, a surface wind has very little effect on the vertical distribution of velocity. However, the ice cover affects the velocity distribution significantly.

In general, velocity components on a plane normal to the streamwise direction are not zero, and they form the secondary currents. Secondary currents are generated by the two separate mechanisms. One is due to the centrifugal force (secondary currents of Prandtl's first kind), and the other is due to the non-homogeneity and non-isotropy of turbulence (secondary currents of Prandtl's second kind). The secondary currents of the first kind have a magnitude of 20-30% of the mainstream velocity, while those of the second kind much smaller (Nezu and Nakagawa, 1993).

The figure below shows the pattern of the secondary currents in a rectangular channel. In the vicinity of the sidewall, two vortices are present. That is, they are the free surface vortex and bottom vortex, rotating in the clockwise and counter-clockwise directions, respectively. In the region away from the sidewall, the secondary currents are not present, which makes the flow truly two-dimensional.

The direction of the flow near the sidewall can easily be understood by applying the Bernoulli theorem. Consider two points A near the sidewall and B away from the sidewall at the free surface. Since the streamwise velocity at A is lower than that at B. Thus, the pressure at A is higher than that at B. This induces the movement of water particle from A to B along the free surface.

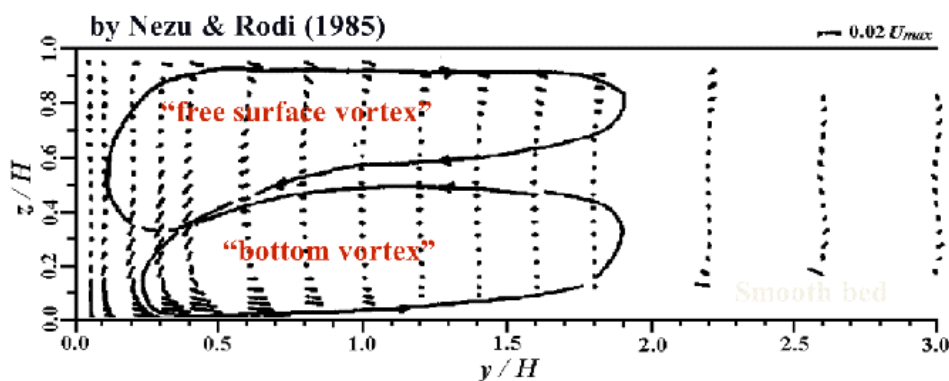


Figure 5. Secondary currents in a rectangular channel (Nezu and Rodi, 1985)

The figure above suggests that a 2D flow can be made in the central region if the channel is wide. Experiments indicate 2D flows exist in the central region when the width is greater than

5 -10 times the flow depth. Thus, a wide open channel can safely be defined if the width is greater than 10 times the flow depth.

8. Velocity-distribution Coefficients

As a result of non-uniform velocity over a channel section, the velocity head of an open-channel flow is larger than $V^2/2g$ (here, V is the mean velocity). In the energy equation, the true velocity head should be given by $\alpha V^2/2g$, where α is the energy correction factor or Coriolis coefficient. The value of α lies between 1.03 and 1.36 for fairly straight prismatic channels.

The non-uniform distribution of velocity also affects the computation of momentum in open-channel flow. Similarly, the momentum is larger than ρQV . The momentum of the fluid passing through a channel section per unit time is given by $\beta \rho QV$. Here, β is the momentum correction factor or Boussinesq coefficient. The value of β lies between 1.01 and 1.12 for fairly straight prismatic channels.

If v is the actual velocity distribution, then the energy correction factor and momentum correction factor are defined by, respectively,

$$\alpha = \frac{\int v^3 dA}{V^3 A} \quad (3)$$

$$\beta = \frac{\int v^2 dA}{V^2 A} \quad (4)$$

9. Pressure Distribution in a Channel Section

If the streamlines have neither substantial curvature nor divergence, the flow is the parallel flow. Otherwise it is the curvilinear flow. Considering the dimensional disparity of the stream, it is reasonable to think the hydrostatic pressure distribution is valid for the gradually varied flow as well as the uniform flow.

In a convex flow, the centrifugal force is acting upward against the gravity, the resulting pressure is less than otherwise hydrostatic pressure of a parallel flow. In a concave flow, the centrifugal force is acting downward reinforcing the gravity, the resulting force is greater than otherwise hydrostatic pressure of a parallel flow. In the figure below, h is the piezometric head, h_s is the hydrostatic head, and c is the pressure head correction for curvature.

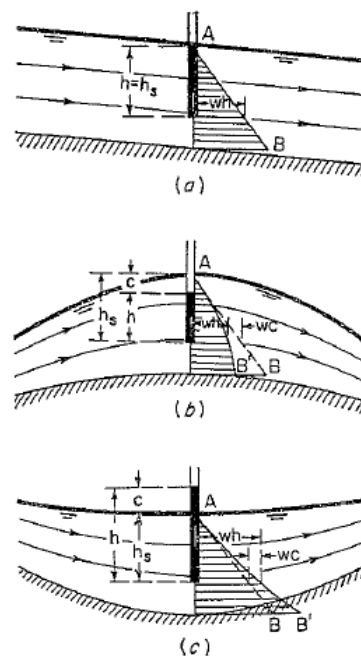


Figure 6. Pressure distribution for (a) parallel flow, (b) convex flow, and (c) concave flow

10. Slope Effect on Pressure Distribution

The pressure is due to the cosine component of the weight of the water column normal to the bed. That is,

$$P = wy \cos \theta dLB \times \cos \theta$$

where B is the channel width. Therefore,

$$h = p / w = y \cos^2 \theta \quad (4)$$

or

$$h = d \cos \theta \quad (5)$$

Apparently, if the angle θ is small, this factor will not differ appreciably from unity. In fact, the correction decreases the pressure head by an amount less than 1% until θ is nearly 6° , a slope of about 1 in 10. Since the slope of actual channels is much less than 1 in 10, the slope effect can safely be ignored.

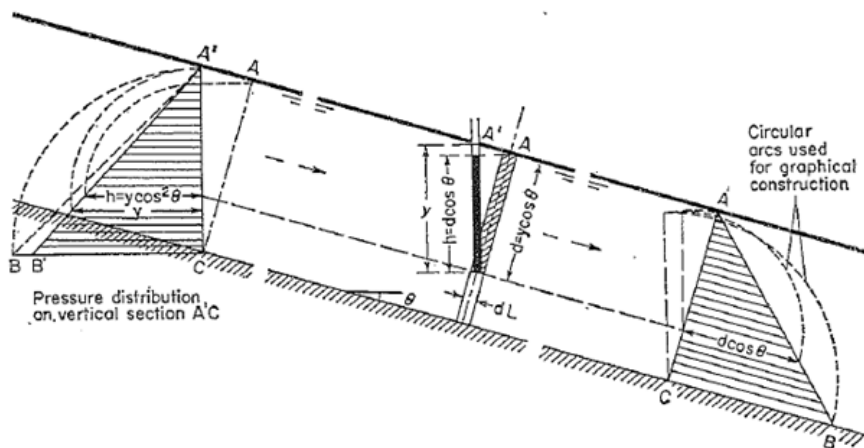


Figure 7. Pressure distribution in parallel flow in channels of large slope