Scalar flux modeling of solute transport in open channel flows: Numerical tests and effects of secondary currents

Modélisation du transport scalaire d’un soluté en canal: tests numériques et effets des courants secondaires

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ABSTRACT
Numerical experiments involving various algebraic scalar flux models for solute transport in open channel flows are presented. Five algebraic scalar flux models including those of Daly and Harlow, Abe and Suga, Suga and Abe, Sommer and So, and Wikstrom et al. are tested. For the flow computation, a Reynolds stress model is used. The models are applied to laboratory experiments of solute transport in rectangular and compound open channel flows. The performance of each model is evaluated both qualitatively and quantitatively. It is found that Daly and Harlow’s model, although simple, predicts the solute transport most accurately. Further, with reference to the simulation results, the roles of the Reynolds fluxes and secondary currents in the solute transport equation are investigated. It is found that the Reynolds fluxes and secondary currents reduce and move the peak concentration, respectively.

RéSUMÉ
Des expériences numériques impliquant différents modèles de transport scalaire algébrique pour le transport de soluté en canal sont présentées. Cinq modèles de transport scalaire algébriques, comprenant ceux de Daly et Harlow, Abe et Suga, Suga et Abe, Sommer et So, et Wikstrom et al. sont testés. Pour le calcul du débit, un modèle de contraintes de Reynolds est utilisé. Les modèles sont appliqués aux expériences de laboratoire de transport de soluté en canal rectangulaire composé. Les performances de chaque modèle sont évaluées à la fois qualitativement et quantitativement. Il se trouve que le modèle de Daly et Harlow, bien que simple, prédit le transport de soluté avec le plus de précision. En outre, dans les résultats de simulation, les rôles de transfert de Reynolds et de courants secondaires dans l’équation de transport de soluté sont examinés. Il est constaté qu’ils réduisent et déplacent la concentration maximale, respectivement.

Keywords: Open channel flow, Reynolds flux, Scalar flux model, Secondary currents, Solute transport

1 Introduction
Solute transport in rivers is an important topic in environmental hydraulics. Relevant issues include control discharges of sewage and wastewater, accidental release of contaminants, transport of non-point sources, and waste heat discharge from power plants. Turbulence plays a key role in their mixing in water environments. Thus, to predict the fate of solute or heat, accurate modeling or analysis of solute transport in turbulent flows is required.

Numerical simulations of momentum transport in turbulent open channel flows have been carried out successfully by means of Reynolds-Averaged Navier-Stokes (RANS) models (Pezzinga 1994, Sofialidis and Prinos 1999, Kang and Choi 2006a), Large Eddy Simulations (LES) (Thomas and Williams 1995, Li and Wang 2000, Zedler and Street 2001, Hinterberger et al. 2007, or John and Williams 2008), and Direct Numerical Simulations (DNS) (Joung and Choi 2008). However, turbulence modeling of solute transport in turbulent open channel flow has only rarely been conducted. Djordjevic (1993) presented a two-dimensional (2D) depth-averaged numerical model based on the operator-splitting approach, to solve numerically solute transport in steady open channel flow. He obtained good agreement between measurements and simulation results using a Schmidt number of unity for both the streamwise and lateral directions. Nokes and Hughes (1994) introduced a three-dimensional (3D) semi-analytical model for solute transport in compound...
open-channel flow. Their model requires input data of streamwise mean velocity and distribution of diffusivity for numerical computations.

For non-uniform flows over a channel section, Prinos (1992) and Simoes and Wang (1997) used the linear \(k-\varepsilon\) and the mixing length models, respectively. However, because these models are isotropic turbulence closures, the impact of secondary currents could not be studied. Lin and Shiono (1995) used both the linear and non-linear \(k-\varepsilon\) models for computations and observed that the latter predicts the mean concentration better for the compound channel flow than does the linear model. Shiono et al. (2003) applied the non-linear \(k-\varepsilon\) model to the same problem to investigate the impact of secondary flows on solute transport. They found that secondary currents move the peak concentration point in compound open-channel flows.

A common hypothesis of practical engineering is Reynolds’ analogy, which assumes that eddy diffusivity is the same as eddy viscosity. It is not easy to confirm this in the laboratory though, owing to the difficulty of measuring flow and solute concentration simultaneously. However, if accurate turbulence models for flow and mixing are employed, this assumption can be confirmed using numerical computations. This motivated the current study.

In general, three approaches are available to model scalar flux in the numerical analysis of the solute transport equation, namely the eddy diffusivity model, the algebraic scalar flux model, and the transport equation model for scalar flux. The first approach entails the eddy diffusivity concept to estimate the scalar flux \(u'_c\) as

\[
\overline{u'_c c'} = -\frac{\nu_t}{\sigma_t} \frac{\partial c}{\partial x_i}
\]

(1)

where \(\overline{u'_c c'}\) is the turbulent diffusion flux, \(\nu_t\) is the eddy viscosity, \(\sigma_t\) is the turbulent Schmidt number, and \(\overline{c'}\) is the mean concentration. This simple approach based on the eddy diffusivity concept is unable to predict realistic values of all scalar fluxes because it estimates the scalar flux aligned with the mean scalar gradient. Moreover, \(\sigma_t\) in Eq. (1) is uncertain. Researchers, through numerical studies, have suggested \(\sigma_t\) values ranging between 0.5 and 1.0 (Djordjevic 1993, Lin and Shiono 1995, Simoes and Wang 1997).

The second approach using algebraic expressions for Reynolds-averaged turbulent scalar fluxes has been widely used in engineering applications because it provides moderately accurate mean scalar distributions despite the simplicity of the equations. Daly and Harlow (1970) proposed a simple model based on the generalized gradient diffusion hypothesis. It has been applied to many engineering problems (Abe 2006, Abe and Suga 2001b, Suga and Abe 2000, Peng and Davidson 1999, Rokni and Gatski 2001, Rokni and Sunden 1998). However, the models as proposed by Daly and Harlow is known to underestimate the streamwise component of the scalar flux \(\overline{u'_c c'}\), even for simple shear flows. This motivated to develop various algebraic scalar flux models (Jones and Musonge 1988, Yoshizawa 1988, Sommer and So 1995, Abe and Suga 2001b, Wikstrom et al. 2000). However, most of these models were developed for internal flows without a free surface.

In the transport equation model, the scalar flux is obtained by solving the transport equation

\[
\frac{\partial u'_c c'}{\partial t} = D_{ic} + P_c + B_c + \Pi_{ic} - \varepsilon_{ic}
\]

(2)

where \(D_{ic}\) is the turbulent diffusive transport, \(P_c\) is the mean field production, \(B_c\) is the buoyancy production, \(\Pi_{ic}\) is the pressure-scalar gradient correlation, and \(\varepsilon_{ic}\) is the viscous dissipation. All of the terms, except for the production terms \(P_c\) and \(B_c\), require models. Moreover, transport equations for \(\overline{c^2}\) and \(\varepsilon_{ic}\) have to be solved through additional modeling of the diffusive transport term of \(\overline{c^2}\). This approach is the most accurate if the second-order closure model is used in the flow analysis. However, there are 15 transport equations to be solved (ten for flow and five for scalar fluxes), even for 3D uniform flows, rendering this approach expensive.

Various sub-models have been proposed for pressure-scalar gradient correlation \(\Pi_{ic}\) in Eq. (2). Examples include Shih and Lumley (1985), Craft (1991), and Shikazono and Kasagi (1996). However, these models were developed only for inner flows. Moreover, it has been reported that the models yield poor predictions if applied to other, more complicated flows (Lien and Leschziner 1993, Murakami et al. 1993). An alternative is the algebraic scalar flux model, a model falling in-between the eddy diffusivity model and the transport equation model for scalar flux, providing accuracy at a reasonable computational cost.

The purpose of this study is to propose an algebraic scalar flux model for numerical simulations of solute transport in open channel flows. Numerical experiments were carried out with five algebraic scalar flux models. The Reynolds stress model, a second-order closure model, was used to compute the flow. The models were applied to experiments of solute transport in rectangular and compound channel flows, and the simulation results are compared with the test data. The impact of secondary currents on solute transport is also discussed. Through an analysis of the transport equation using the simulated data, the roles of the secondary currents and the Reynolds fluxes in solute transport are investigated.

2 Numerical models

2.1 Flow model

Steady open-channel flow at a high Reynolds number is considered by assuming uniform flow in the streamwise direction. The RANS equations for incompressible fluids are

\[
\frac{\partial \overline{u_i}}{\partial t} = 0
\]

(3)

\[
\frac{\partial \overline{u_i u_j}}{\partial x_j} + u'_i \frac{\partial \overline{u_i u_j}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \overline{u_i u_j}}{\partial x_j} \right) - \frac{\partial \overline{u'_i u'_j}}{\partial x_j} + g_i
\]

(4)

where \(\overline{u'_i u'_j}\) and \(u'_i = \text{mean and fluctuating velocity components in the } i\text{-direction, respectively.} \)
The Reynolds stress in Eq. (4) is obtained by solving the transport equations for Reynolds stress \( \overline{\mathbf{R}_{ij}} = -\overline{\mathbf{u}_i \mathbf{u}_j} \) such that

\[
\frac{\partial \mathbf{R}_{ij}}{\partial t} + \overline{\frac{\partial \mathbf{R}_{ij}}{\partial x_k}} = -\left( \left( R_{ik} \frac{\partial \mathbf{R}_{kj}}{\partial x_k} + R_{kj} \frac{\partial \mathbf{R}_{ik}}{\partial x_k} \right) + D_{ij} - \epsilon_{ij} + \Pi_{ij} \right)
\]

(5)

where \( D_{ij} \) is transport of \( \mathbf{R}_{ij} \) by diffusion, \( \epsilon_{ij} \) is rate of dissipation of \( \mathbf{R}_{ij} \), and \( \Pi_{ij} \) is transport of \( \mathbf{R}_{ij} \) due to turbulent pressure-strain interactions. Herein, Mellor and Herring’s (1973), Speziale et al.’s (1991), and Rotta’s (1951) models are used for the turbulent diffusion term, the pressure-strain correlation term, and the dissipation rate term, respectively. To account for the damping effect at the free surface, the combination model of Shir (1973) and Gibson and Launder (1978) was added to the pressure-strain term. This choice of sub-model is based on the numerical experiments reported by Choi and Kang (2001). With these sub-models, Kang and Choi (2006a, b) successfully simulated the mean flow and turbulence statistics of rectangular and compound open-channel flows.

2.2 Solute transport model

The Reynolds-averaged transport equation for a passive scalar and the WWJ model of Wikstrom Suga and Abe (2000), the SS model of Sommer and So (1995), the AS model of Abe and Suga (2001b), the SA model of Abe and Suga (2001a), and Rogers et al.’s (1989) models were used, namely the DH model of Daly and Harlow (1970), the AS model of Abe and Suga (2001b), the SA model of Suga and Abe (2000), the SS model of Sommer and So (1995), and the WWJ model of Wikstrom et al. (2000).

**DH model**

Daly and Harlow (1970) introduced an eddy diffusivity tensor proportional to the Reynolds stress, and proposed the following DH model for the scalar flux

\[
\overline{u_i' c'} = -C_c \tau_i \overline{u_i' u_j} \frac{\partial \overline{c'}}{\partial x_j}
\]

(7)

where \( C_c (= 0.22) \) = model constant and \( \tau_i (= k/\varepsilon) \) = characteristic time, with \( k \) = turbulent kinetic energy and \( \varepsilon \) = dissipation rate of \( k \).

**AS model**

Kim and Moin (1989), pointing out that the scalar fluctuation \( c' \) in the wall shear region is stronger correlated with the streamwise \( u' \) than with the wall normal velocity fluctuation, suggested proportionality between the scalar flux and the Reynolds stress as

\[
\overline{u_i' c'} \propto \overline{u_i u}\]

(8a)

and

\[
\overline{c'c'} \propto \overline{v'v}\]

(8b)

As the DH model cannot satisfy the above proportionality, Abe and Suga (2001b) proposed for the scalar flux

\[
\overline{u_i' c'} = -C_c \tau_i \overline{u_i' u_j} \frac{\partial \overline{c'}}{\partial x_j}
\]

(9)

where \( C_c (= 0.45) \) = model constant. Abe (2006) compared the DH model with the AS model by computing both plane channel flow and plane impinging jet. Through comparisons with the DNS and LES data, Abe found that both models predict the mean concentration and vertical component of the scalar flux well. However, he found that the DH model under-estimates the streamwise component of the scalar flux.

**SA model**

Suga and Abe (2000) proposed a turbulent heat flux model for channel flows with and without a free surface. They expressed the eddy diffusivity tensor in nonlinear Reynolds stress terms, resulting in the heat flux model

\[
\overline{u_i' c'} = -C_0 k \tau_i (\sigma_{ij} + \alpha_{ij}) \frac{\partial \overline{c'}}{\partial x_j}
\]

(10)

where

\[
\sigma_{ij} = C_{c_0} \delta_{ij} + C_{c_1} \overline{u_i u_j} k + C_{c_2} \overline{u_i u_j u_i u_j} k^2
\]

(11a)

\[
\alpha_{ij} = C_{c_0} \tau \Omega_{ij} + C_{c_1} \left( \overline{u_i u_j} k + \Omega_{ij} - \frac{\overline{u_i u_j}}{k} \right)
\]

(11b)

with the rotation tensor \( \Omega_{ij} \) defined by

\[
\Omega_{ij} = \frac{1}{2} \left( \frac{\partial \overline{u_i}}{\partial x_j} - \frac{\partial \overline{u_j}}{\partial x_i} \right)
\]

(11c)

In Eqs. (10) and (11), \( C_0, C_{c_1}, C_{c_2}, C_{c_0}, C_{c_1}, \) and \( C_{c_2} \) are the model parameters.

Suga and Abe (2000) applied their model to plane channel, open channel, and plane Couette-Poiseuille flows and showed that the model accurately simulates these flows through comparisons with the DNS data by Abe and Suga (2001a) and the LES data by Abe and Suga (2001a), Rogers et al. (1989), and Kim and Moin (1989). However, due to the uncertainty of the model parameters, Suga and Abe (2000) obtained inaccurate scalar fluxes near the free surface for an open-channel flow at \( \text{Re} (= U_b H/\nu) = 2,800 \) with \( U_b = \) bulk flow velocity.

**SS model**

Assuming similarity between the transport of \( \overline{u_i' c'} \) and that of turbulent kinetic energy, Sommer and So (1995) proposed for the scalar flux

\[
\overline{u_i' c'} = k \tau_m \frac{1}{C_{c_1}} [-C_{c_2} \delta_{ij} + 2 C_{\mu} \tau_i S_{ij} + (1 - C_{c_2}) C_{\lambda} \tau_m (S_{ij} + \Omega_{ij})] \frac{\partial \overline{c'}}{\partial x_j}
\]

(12)

with \( C_{\lambda} (= 0.095), C_{\mu} (= 0.096), C_{c_1} (= 3.28), \) and \( C_{c_2} (= 0.4) \) are the model constants, \( \tau_m \) = characteristic time given by
\[ \tau(2R)^{0.5} \] with \( R = \frac{\tau_l}{\tau_c} = \text{time scale ratio} \) and \( \tau_c \) = \( c^2/\varepsilon \) = time scale for scalar field, and \( S_{ij} \) = rate of the strain tensor given by

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{13}
\]

Since the SS model cannot account for the effect of turbulence anisotropy, Abe et al. (1996) proposed a value of \( C_\lambda = 0.095 \), which is used herein, rather than the analytical value of \( 2/(3C_{\lambda}) = 0.203 \).

**WWJ model**

By neglecting both advection and diffusion in non-dimensional scalar flux for nearly homogeneous steady flows, Wikstrom et al. (2000) proposed as scalar flux model

\[
\overline{u_i^c c^2} = -(1 - C_{c4})B_{ij} \frac{k}{\varepsilon} \frac{\partial \tau}{\partial x_k} \tag{14}
\]

\[
(G^2 - \frac{Q_1}{2})I - G(C_s S_{ij} + C_\Omega \Omega_{ij}) \frac{\varepsilon}{2}
\]

\[
B_{ij} = \frac{+ (C_s S_{ij} + C_\Omega \Omega_{ij}) \frac{\varepsilon}{2}}{G^3 - \frac{1}{2} G Q_1 + \frac{1}{4} Q_2}
\]

\[
G = \frac{1}{2} \left( C_{c4} - 1 - \frac{1}{R} + \frac{P_k}{\varepsilon} \right). \tag{16a}
\]

\[
Q_1 = c_i^2 H_i + c_\Omega^2 \Omega_i, \tag{16b}
\]

\[
Q_2 = \frac{2}{3} C_{c4} H_{III} + 2 C_s C_\Omega^2 IV \tag{16c}
\]

where \( P_k \) = rate of production of turbulent kinetic energy, \( H_i \) (= \( S_{ij} S_{ji} \)), \( \Omega_i \) (= \( \Omega_{ij} \Omega_{ji} \)), \( III_i \) (= \( S_{ij} \Omega_{jk} S_{ki} \)), and \( IV \) (= \( S_{ij} \Omega_{jk} \Omega_{ki} \)) are the invariants of the mean flow gradients, and \( C_s \) (= \( 1 - C_{c2} - C_{c3} \)), \( C_\Omega \) (= \( 1 - C_{c2} + C_{c3} \)), \( C_{c1} \) (= \( 4.51 \)), \( C_{c2} \), \( C_{c3} \), and \( C_{c4} \) are the model constants. Wikstrom et al. (2000), on the basis of numerical experiments, proposed \( C_{c2} = C_{c3} = C_{c4} = 0 \) for various flows. The WWJ model is similar to the DH model if \( B_{ij} = 1.0 \) in Eq. (14). Wikstrom et al. (2000) applied the WWJ model to homogeneous shear flow, turbulent channel flow, and flow behind a heated cylinder, and found, through comparisons with the DNS data of Rogers et al. (1986), that their model successfully simulates these flows.

### 3 Numerical tests of scalar flux models

To investigate the performance of the algebraic scalar flux models, numerical tests were carried out by applying the models to the two experimental data sets of Kearney (2000) and Shiono et al. (2003). The detailed experimental conditions are shown in Fig. 1. In experiments on rectangular open-channel flow, a flow depth of \( H = 0.085 \) m and a channel width of \( B = 1.2 \) m were used. The resulting width-to-depth ratio was 14.1, and the channel slope was 0.0005. The injection point of the dye was located near the free surface at \( y = 0.085 \) and \( z = 0.082 \) m in Case S1 and \( y = 0.17 \) m and \( z = 0.082 \) m in Case S2. In experiments on compound-open-channel flow, the flow depths of the main channel and floodplain were \( H = 0.11 \) m and \( h = 0.055 \) m with respective widths of \( B = 0.2 \) m and \( b = 0.1 \) m. The injection point was located also near the free surface at \( y = 0.1 \) m and \( z = 0.108 \) m in Case C1 and \( y = 0.15 \) m and \( z = 0.108 \) m in Case C2. Rhodamine 6G at a concentration of 2,500 ppb was applied, and Laser Doppler anemometer (LDA) and Laser Induced Fluorescence (LIF) were employed to measure the instantaneous velocity and concentration components simultaneously. The measuring section was located 1 m downstream of the injection point.

#### 3.1 Secondary currents and streamwise mean velocity

Figure 2 shows a contour plot of the streamwise mean velocity and secondary current vectors in the wall region for rectangular open-channel flow. The simulation results by the present model are successfully compared with the measurement data of Shiono.
and Feng (2003). Note that both the free surface and bottom vortices are present in the wall vicinity, bulging the contour lines to the upper and lower corners, respectively. In the plots, the velocity dip induced by secondary currents, is observed in both the simulated and the measurement data.

Figure 3 shows the same plot for compound open-channel flow. Note that the simulation results are in good agreement with the measurement data of Shiono and Feng (2003). Two velocity maximums are present in both the main channel and the floodplain, resulting in a shear layer at the interface. In addition, both the simulated and the measurement data show that counter-rotating twin vortices are generated at the junction of main channel and floodplain. These vortices bulge the isovels to the upper center in the main channel.

The vertical distribution of streamwise mean velocity at various lateral locations is plotted in Fig. 4. For rectangular open-channel flow, a fairly good agreement is observed between the simulation results and the measurement data of Kearney (2000). For compound channel flow, good agreement is also evident except for the interfacial region close to the bottom wall, i.e. at \( y = 0.1 \text{ m} \).

### 3.2 Solute transport in rectangular channel flow

Figure 5 shows for rectangular channel flow the lateral distributions of mean concentration and scalar fluxes \( \bar{v}c' \) and \( \bar{w}c' \) at three heights. The simulation results from the five models are compared with the measurement data. Regarding the mean concentration in Fig. 5(a), it is found that the AS model slightly over-predicts the peak concentration at \( z = 0.08 \text{ m} \). At \( z = 0.07 \text{ m} \), the SA and the WWJ models under-predict the peak concentration. Interestingly, the SA model incorrectly predicts the point of peak concentration at both \( z = 0.08 \text{ m} \) and \( z = 0.07 \text{ m} \). At \( z = 0.06 \text{ m} \), the SS model
Similarly, the AS model over-predicts the peak of the scalar flux at \( z = 0.07 \) m. At \( z = 0.06 \), the accuracy of neither model can be discerned clearly, due to scatter of the test data.

Figure 6(a) shows the vertical distribution of the mean concentration at various lateral locations. In Fig. 6(a), the measurement data of Kearney (2000) at \( y = 65 \) mm and 85 mm show the general trend that the mean concentration has the maximum immediately below the water surface and decreases toward the bottom, due to the secondary currents near the sidewall (Fig. 2a). Only the SS model is observed to successfully reproduce this feature. However, the mean concentration computed from the other models decreases monotonically toward the bottom. Specifically, at \( y = 105 \) mm, the SA and the AS models over-estimate the mean concentration seriously near the free surface.

The vertical distribution of scalar flux \( w/c' \) shown in Fig. 6(b) has the peak immediately below the free surface and decreases toward the bottom. At \( y = 65 \) mm and 85 mm, the SA model incorrectly simulates the value and location of the peak scalar flux. The other models predict the scalar flux at a similar accuracy, i.e. they over-predict below the point of the peak scalar flux.

### 3.3 Solute transport in compound channel flow

Figure 7(a) shows the lateral distribution of the mean concentration at various heights for compound channel flow. The results computed by the various models are compared with the test data of Shiono et al. (2003). Both the simulation results and the measurement data indicate that the peak concentration is located in the floodplain, slightly shifted from the interface between main channel and floodplain. This is a direct effect of the secondary currents shown in Fig. 3(a). Note that the measured concentration is not zero in the main channel, the simulated profiles having a gradual decay from the peak to the left sidewall. This seems to be caused by measurement errors in the tests. All the models predict the mean concentration with similar accuracies over the height.

Figure 7(b) shows the lateral distribution of the scalar flux \( w/c' \) at three heights. Similarly to rectangular channel flow, the SA model appears to over-predict the peak scalar flux at \( z = 0.105 \) m. At \( z = 0.09 \) m and \( z = 0.075 \), the scalar flux profiles simulated by all models appear to be similar. All of the models over-predict at \( z = 0.09 \) and again at \( z = 0.075 \) m, especially for the floodplain in the latter case.

Figure 8(a) shows the vertical distribution of the mean concentration at various lateral locations. Although the measurement data are severely scattered, the plots clearly indicate that all of the five models under-predict the mean concentration at \( y = 0.075 \) m and 0.1 m. At \( y = 0.125 \) m, all models have similar predictions. In general, the predicted mean concentration is incorrect near the free surface specifically, under-estimated at \( y = 0.075 \) m and \( y = 0.1 \) m and over-estimated at \( y = 0.125 \) m.

The vertical distribution of scalar flux \( w/c' \) is given in Fig. 8(b). The measurement data have a severe scatter at \( y = 0.075 \) m and \( y = 0.125 \) m, making model tests difficult. At \( y = 0.075 \) m, all five models predict similar values. A close agreement between the measurement data and the profiles computed by the DH and SS models is seen at \( y = 0.1 \) m, whereas the SA model over-predicts...
Figure 6  Vertical distribution of mean concentration and scalar flux in rectangular channel for Case S1 (a) mean concentration, (b) scalar flux $w'c'$

Figure 7  Lateral distribution of mean concentration and scalar flux in compound channel for Case C1 (a) mean concentration, (b) scalar flux $w'c'$

Figure 8  Vertical distribution of mean concentration and scalar flux in compound channel for Case C1 (a) mean concentration, (b) scalar flux $w'c'$
the scalar flux. At \( y = 0.125 \) m, all models except SS predict similarly. The scalar flux computed by the SS model appears to be different from the others near the free surface.

### 3.4 Accuracy of scalar flux models

The previously simulated profiles revealed the general features of data are calculated. The mean discrepancy ratio is defined as the discrepancy ratios between the prediction and measurement:

\[
Me = 10^b
\]

where

\[
b = \frac{1}{N} \sum \log \left( \frac{\Phi_{\text{comp}}}{\Phi_{\text{meas}}} \right)
\]

In Eq. (17), \( Me = \) discrepancy ratio, \( N = \) number of data, and \( \Phi_{\text{comp}} \) and \( \Phi_{\text{meas}} = \) computed results and the measurement data, respectively. A smaller value of discrepancy ratio reflects a closer agreement between the computed results and the measurement data.

Table 1 lists the computed discrepancy ratios. The first three columns include the discrepancy ratios from the lateral distributions, and the next two columns those from the vertical distributions. The sixth columns list the arithmetic means of the five ratios. For rectangular channel flows, the DH, SA and AS models appear to predict solute transport accurately. For compound channel flows, the discrepancy ratios for the DH and SA models are small in both Cases C1 and C2. Over the four tests, the arithmetic means for the SA and DH models are 2.11 and 2.19, respectively, indicating that these models perform better than the others.

The quantitative results listed in Table 1 suggest that both the DH model and the SA model are accurate. The comparisons in Sections 3.2 and 3.3 indicate, moreover, that the DH model is capable of accurately predicting the mean concentration and scalar fluxes. However, although it is true that the discrepancy ratio for the SA model is the smallest, the SA model incorrectly predicts the location of peak concentration and over-predicts the scalar flux (Fig. 5). Moreover, the SA model is more complicated than the DH model, suggesting that the latter is the best choice for predicting solute transport in open-channel flows.

### 4 Impact of secondary currents

Figure 9 shows simulated contour plots of the mean concentration, together with secondary current vectors for Case S1 at various longitudinal locations. Herein, the DH model is used for computation. A gradual dilution in the streamwise direction of dye injected at \( x = 0 \) is seen. At \( x = 3 \) m, the dye reaches the sidewalls as well as the bottom, showing contour lines bulged to the lower left corner, whereas at \( x = 1 \) m the concentration distribution is almost symmetric. This asymmetry is due to secondary currents formed for \( 0.5 < z/H < 0.8 \), where the free surface and the bottom vortices merge and head to the left sidewall. The secondary currents also affect the point of the peak concentration, shifting it slightly to the right; however, their impact is hardly noticeable.

Figure 10 shows the same contour plots for the compound channel flow (Case C1). Compared with Fig. 9, a shift of the peak concentration point is observed. It appears that the free surface vortex in the main channel and the twin vortex at the junction make the contour lines at \( x = 3 \) m bulged toward the main channel. Similarly, the secondary currents on the floodplain make the contour lines bulged to the right sidewall. Compared with rectangular channel flow, the impact of the secondary currents is pronounced. This is not because of the magnitude of the secondary currents but because of the location of the injection point. That is, the injection point in compound channel flow is located in the region of the strong secondary currents. For
compound channel flow, the simulated maximum magnitude of the secondary current vectors is 2.9% of the maximum velocity, which is not significantly higher than 2.07% for rectangular channel flow.

Figure 11(a) shows the lateral distribution of the mean concentration at \( z = 0.08 \text{ m} \) for rectangular channel flow. The location of the peak concentration at \( x = 0.5 \text{ m} \) is slightly shifted to the right due to secondary currents; however, the impact is hardly noticeable. The location of the peak concentration is no longer moving beyond \( x = 1.0 \text{ m} \). In general, the shape of the mean concentration observed is of Gaussian distribution. However, the distribution beyond \( x = 3.0 \text{ m} \) shows the left tail thicker than the right (Fig. 9).

For compound channel flow, the mean concentration at \( z = 0.105 \text{ m} \) is shown in Fig. 11(b). The figure clearly depicts the shift of the location of the peak concentration to the right. A continuous shift is noticeable up to \( x = 2.0 \text{ m} \), beyond which its location hardly moves. The distribution of the mean concentration at \( x = 5 \text{ m} \) becomes quite uniform in the transverse direction. Unlike for rectangular channel flow, the mean concentration shows a distribution skewed to the left, rather than a Gaussian distribution, which is due to secondary currents. That is, the secondary currents make the mean concentration in the floodplain higher than in the main channel.

5 Solute transport rate

For steady uniform flows, the scalar transport equation without molecular diffusion takes the form

\[
\frac{\partial uc}{\partial x} = - \left( \frac{\partial \nu c}{\partial y} + \frac{\partial \omega c}{\partial z} \right) - \frac{\partial uc}{\partial y} - \frac{\partial uc}{\partial z} \tag{18}
\]

Using measured data, Shiono and Feng (2003) investigated the impacts of Reynolds flux and secondary currents on the solute transport rate. However, they ignored the vertical component of the secondary currents, the fourth term on the right side of Eq. (18) due to test resolution. Herein, a similar analysis is performed using simulated data, including the vertical component.

Figure 12(a) shows the lateral distribution of the magnitude of each term on the right side of Eq. (18) for rectangular channel flow. It is found that at \( z = 80 \text{ mm} \), the location of the minimum value of \( \partial \nu c / \partial y \) or \( \partial \omega c / \partial z \) is similar to that of the peak of the mean concentration. At this height, the direction of the secondary currents is from left to right. This suggests that these scalar fluxes reduce the mean concentration peak in the streamwise direction. Specifically, \( \partial \omega c / \partial z \) reduces the mean concentration over the entire width. In contrast, \( \partial \nu c / \partial y \) reduces the peak concentration in the middle of the channel, while it increases the concentration at the tails. Additionally, the vertical component of the secondary currents \( \partial uc / \partial z \) decreases the mean concentration over the entire width; however, its effect is small compared with the scalar fluxes. It is also noteworthy the lateral component of the secondary currents \( \partial uc / \partial y \) reduces the mean concentration on the left side of the injection point, while it increases the mean concentration on
Figure 10 Mean concentration contours with secondary current vectors in streamwise direction in compound channel for $x = (a) \ 1.0 \ m$ (measuring section), (b) 3.0 m, (c) 4.0 m, (d) 5.0 m, (e) 6.0 m

Figure 11 Mean concentration distribution in streamwise direction for (a) rectangular channel at $z = 0.08 \ m$, (b) compound channel at $z = 0.105 \ m$

the right side. That is, the term $\partial \overline{vc}/\partial y$ moves the peak concentration to the right by increasing the mean concentration in the flow direction of the secondary currents. However, the effect of $\partial \overline{vc}/\partial y$ appears to be smaller than that of the scalar fluxes.

Figure 12(b) shows the same plot for $z = 60 \ mm$, with the secondary currents from right to left. The distribution of $\partial \overline{vc}'/\partial y$ does not change significantly compared with that for $z = 80 \ mm$. However, the roles of the scalar flux term $\partial \overline{wc}'/\partial z$ and the vertical component of the secondary currents $\partial \overline{wc}/\partial z$ are reversed, i.e. they increase the mean concentration in the streamwise direction. Note also that the lateral component of the secondary currents $\partial \overline{vc}/\partial y$ has the opposite trend to that at $z = 80 \ mm$. This term thus increases the mean concentration in the flow direction of the secondary currents by shifting the peak concentration to the left sidewall.

Figure 13 shows the contributions of each term in Eq. (18) for compound channel flow. Similar to rectangular channel flow, the scalar flux term $\partial \overline{wc}'/\partial z$ is found to reduce the mean concentration over the entire width; however, its effect is smaller than for rectangular channel flow. Also, $\partial \overline{vc}'/\partial y$ plays a key role in
the left side. The effect, although similar for rectangular channel flow, appears to be significant, indicating that the shift of the peak concentration is due to the lateral component of the secondary currents. The vertical component of the secondary currents $\partial \overline{c} / \partial z$, opposite to $\partial \overline{c} / \partial y$, is found to affect the mean concentration less than the lateral component.

6 Conclusions

Numerical experiments of various algebraic scalar flux models to numerically simulate solute transport in open-channel flows are presented. Five algebraic scalar flux models including Daly and Harlow’s, Abe and Suga’s, Suga and Abe’s, Sommer and So’s, and Wikstrom et al.’s were tested. The models were applied to laboratory measurements of solute transport in the rectangular and compound open-channel flows. To compute the flow, the Reynolds stress model of Kang and Choi was used.

For solute transport in rectangular channel flow, simulated lateral distributions of the mean concentration and scalar fluxes were compared with measured data. The results suggested that the performance of the DH model is best and those of the SS and WWJ models are moderate. For solute transport in compound channel flow, the DH, SS, and WWJ models predict the lateral distributions of the mean concentration and scalar flux rather accurately, though the measurement data showed severe scattering. With regard to the vertical distributions, the predictions with the DH and WWJ models were relatively accurate. The discrepancy ratio between predicted and measured profiles also supported that the DH model is best for numerical simulations of solute transport in open-channel flows.

On the basis of the mean concentration profiles simulated by the DH model, the impact of the secondary currents on the solute transport was investigated. It was observed that the secondary currents in the compound channel flow clearly move the location of peak concentration to the floodplain. This resulted in a skewed distribution of the mean concentration in the streamwise direction. A similar but significantly weak impact was noticed in the rectangular channel. This is because the dye was injected into the region where the secondary currents are weak, not because the magnitude of the secondary currents is weaker in the rectangular than that in the compound channel flow.

With reference to the simulation results, the roles of the Reynolds flux and the secondary currents in solute transport were investigated. For both rectangular and compound channel flows, it was found that the Reynolds fluxes reduce the peak and thicken the tails of the mean concentration. It also was discovered that the secondary currents affect the magnitude of the mean concentration over the entire width, moving the peak concentration in the flow direction. However, their impact was weak compared with that of the Reynolds fluxes.

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**Notation**

- \( B \) = Channel width
- \( B_c \) = Buoyancy production
- \( \bar{\varepsilon}, \bar{\varepsilon}' \) = Mean and fluctuating components of the concentration
- \( C \) = Model constants
- \( D_d \) = Turbulent diffusive transport
- \( D_{ij} \) = Transport of \( R_{ij} \) by diffusion
- \( g \) = Gravitational acceleration
- \( H \) = Flow depth
- \( k \) = Turbulent kinetic energy
- \( Me \) = Discrepancy ratio
- \( N \) = Number of data
- \( \overline{\nabla} \) = Mean pressure
- \( P_c \) = Mean field production
- \( P_h \) = Rate of turbulent kinetic energy production
- \( R \) = Time scale ratio
- \( S_{ij} \) = Rate of strain tensor
- \( u_{ij}^{'} \) = Mean and fluctuating velocity components
- \( \overline{u'^{'}u'^{'}_{ij}} \) = Scalar flux
- \( -u'^{'}_j u'^{'}_i (= R_{ij}) \) = Reynolds stress per unit fluid density
- \( x, y, \) and \( z \) = Longitudinal, transverse, and vertical coordinates

**Greek Symbols**

- \( \varepsilon \) = Dissipation rate of \( k \)
- \( \varepsilon_{ve} \) = Viscous destruction
- \( \varepsilon_{ij} \) = Rate of dissipation of \( R_{ij} \)
- \( \lambda \) = Molecular diffusivity
- \( \nu \) and \( \nu_t \) = Kinematic and eddy viscosity, respectively
- \( \Omega_{ij} \) = Rotation tensor
- \( \Pi_1 \) = Pressure-scalar gradient correlation
- \( \Pi_{ij} \) = Transport of \( R_{ij} \)
- \( \Phi_{comp} \) and \( \Phi_{meas} \) = Computed results and test data
- \( \rho \) = Fluid density
- \( \sigma_t \) = Turbulent Schmidt number
- \( \tau_m \) = Time scale for scalar field
- \( \tau_s \) and \( \tau_c \) = Characteristic time, and
- \( I_{1}, I_{2}, I_{3}, \) and \( IV \) = Invariants of mean flow gradients

**References**


