Lateral distribution of unit discharge in a compound channel with vegetation on the floodplain

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Abstract

This paper presents a calibration of the 1d model for distributing the unit discharge in a compound channel with floodplain vegetation. The 1d model, including the normalized eddy viscosity related with turbulent shear, is based on the momentum equation in the lateral direction. The RANS model is employed to calibrate the normalized eddy viscosity. The RANS equations in the curvilinear coordinates are solved with the non-linear k-ε model. The models are applied to the Severn River in UK. The unit discharges distributed in the compound channel section are obtained and the total discharge is estimated. The eddy viscosity profile obtained by the RANS model is provided and discussions are given.

Keywords: open-channel flow, compound channel, floodplain vegetation, non-linear k-ε model, eddy viscosity

INTRODUCTION

A one-dimensional (1d) model was proposed by Darby and Thorne (1996) to predict the stage-discharge relationship in a channel with vegetated floodplain. The model contains a parameter related with turbulent viscosity that can be determined empirically.

The model is very useful in constructing the stage-discharge curve in the compound channel. Nowadays, most urban streams have compound sections for the use of floodplain and they are likely to have vegetation on the floodplain to enhance the amenity of them. In this regard, since the model is capable of considering vegetation on the floodplain, its impact on the flood carrying capacity of the stream can be assessed. This is crucial to the flood management of urban streams.

The model can also be used to estimate the roughness coefficients in a compound channel. That is, through the best-fit to the observed data, roughness coefficients for the main channel and floodplain can be obtained. Choi and Shin (2009) applied the model to a stream in Korea and revealed that roughness coefficients that are being used are smaller. This indicates an unsafe aspect of the flood management of the stream. In Korea, the roughness coefficients being used in the stream management are estimated by matching simply the 1d backwater computations are observed stage data without consideration of the stage-discharge characteristics of the channel section.

In addition, the model can be used to predict the change in the longitudinal dispersion coefficient in a stream with a consideration of the floodplain vegetation. Perucca et al. (2009) investigated the impact of riparian vegetation on the dispersion of pollutants in a stream. They found that riparian vegetation increases significantly the dispersion coefficient by altering the lateral distribution of the depth-averaged velocity in a channel section.

The purpose of this study is to calibrate the normalized eddy viscosity in the 1d model to distribute the unit discharge laterally in a compound channel. The RANS model with the non-linear k-ε closure is used for the calibration of the parameter. The profiles of the streamwise mean velocity and depth-averaged eddy viscosity are given and discussed.

The 1d model

In order to distribute the unit discharge q in the lateral (y-) direction, the following momentum equation can be solved:

$$ghS = \frac{\tau_b}{\rho} + \frac{F_D}{\rho} - \frac{\partial}{\partial y}\left(\overline{\nu_t}\frac{\partial q}{\partial y}\right) \qquad (1)$$

where g gravitational acceleration, h flow depth, S channel slope, ρ water density, τ_b bottom friction, F_D drag force due to vegetation (per unit area), and $\overline{\nu_t}$ depth-averaged eddy viscosity. The above equation suggests that the driving force due to gravity is balanced by the bottom shear, vegetation drag, and lateral shear.

In Eq.(1), the bottom shear can be expressed in terms of Manning's roughness coefficient n, i.e.,

$$\tau_b = \frac{\rho g n^2}{h^{7/3}} q^2 \qquad (2)$$

The drag due to vegetation in Eq.(1) is given by

$$F_D = \frac{1}{2} C_D \rho A_V U_1^2 \qquad (3)$$

where C_D is the drag coefficient, A_v is the projected area, U_1 is the depth-averaged velocity in the vegetation layer. Huthoff et al. (2007) and Yang and Choi (2009) proposed the following relationship for U_1:

$$U_1 = \sqrt{\frac{2ghS}{aC_D h_1}} \tag{4}$$

where h_1 is the height of vegetation layer. Rrgarding the depth-averaged eddy viscosity $\overline{\nu_t}$, the following relationship is used:

$$\overline{\nu_t} = \alpha U^* h \tag{5}$$

where α is an empirical parameter (or normalized eddy viscosity) and U^* is the shear velocity (= $[gHS]^{0.5}$). In two-dimensional open-channel flow models, a value of $\alpha = \kappa/6$ is commonly used (here, κ is the von Karman constant). This value can analytically be obtained with the help of logarithmic law velocity profile. However, Darby and Thorne (1996) proposed $\alpha = 0.16$, which was obtained through numerical tests with a data set in a river in UK. A discrepancy is present in the use of the normalized eddy viscosity in two approaches.

Non-linear k-ε model

The non-linear k-ε model is applied to the same problem herein. In order to compute the flow over natural boundary, the following transformation mapping the physical space onto a mathematical space is necessary:

$$x = x\,;\quad \xi = \xi(y,z)\,;\quad \eta = \eta(y,z) \tag{6a,b,c}$$

where (y, z) is the physical plane, (ξ, η) is the transformed plane, and x denotes the streamwise direction. Consider a steady open-channel flow at a high Reynolds number, and assume that the flow is uniform in the streamwise direction. Thus, the Reynolds-Averaged Navier Stokes (RANS) equations in generalized curvilinear coordinates can be written as

$$\frac{v^c}{h_1}\frac{\partial u}{\partial \xi} + \frac{w^c}{h_2}\frac{\partial u}{\partial \eta} = -gS_0 - \frac{1}{J}\left[\frac{\partial}{\partial \xi}\left(h_2 \overline{u'v^{c}{'}}\right) + \frac{\partial}{\partial \eta}\left(h_1 \overline{u'w^{c}{'}}\right)\right] \tag{7}$$

$$\begin{aligned}\frac{v^c}{h_1}\frac{\partial v^c}{\partial \xi} + \frac{w^c}{h_2}\frac{\partial v^c}{\partial \eta} + \frac{v^c w^c}{h_1 h_2}\frac{\partial h_1}{\partial \eta} - \frac{w^{c2}}{h_1 h_2}\frac{\partial h_2}{\partial \xi} &= -\frac{1}{\rho h_1}\frac{\partial p}{\partial \xi} \\ &- \frac{1}{J}\left[\frac{\partial}{\partial \xi}\left(h_2 \overline{v^{c}{'}^2}\right) + \frac{\partial}{\partial \eta}\left(h_1 \overline{v^{c}{'}w^{c}{'}}\right)\right] - \frac{\overline{v^{c}{'}w^{c}{'}}}{h_1 h_2}\frac{\partial h_1}{\partial \eta} + \frac{\overline{w^{c}{'}^2}}{h_1 h_2}\frac{\partial h_2}{\partial \xi}\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{v^c}{h_1}\frac{\partial w^c}{\partial \xi} + \frac{w^c}{h_2}\frac{\partial w^c}{\partial \eta} + \frac{v^c w^c}{h_1 h_2}\frac{\partial h_1}{\partial \xi} - \frac{v^{c2}}{h_1 h_2}\frac{\partial h_2}{\partial \eta} &= -\frac{1}{\rho h_2}\frac{\partial p}{\partial \eta} \\ &- \frac{1}{J}\left[\frac{\partial}{\partial \xi}\left(h_2 \overline{v^{c}{'}w^{c}{'}}\right) + \frac{\partial}{\partial \eta}\left(h_1 \overline{w^{c}{'}^2}\right)\right] - \frac{\overline{v^{c}{'}w^{c}{'}}}{h_1 h_2}\frac{\partial h_2}{\partial \xi} + \frac{\overline{v^{c}{'}^2}}{h_1 h_2}\frac{\partial h_1}{\partial \eta}\end{aligned} \tag{9}$$

which are momentum equations in the x-, y-, and z-directions, respectively. In Eqs.(7)-(9), u, v^c, and

w^c are the streamwise, lateral, and vertical mean velocities in the transformed space, h_1 and h_2 are the coordinate transformation scale factors, J is the determinant of Jacobian, p is the pressure, and $\overline{u_i'u_j'}$ is the Reynolds stress.

The non-linear k-ε model by Speziale (1987) estimates the Reynolds stresses in Eqs.(7)-(9) using the following relationship:

$$-\overline{u_i'u_j'} = -\frac{2}{3}k\delta_{ij} + \frac{k^{5/2}}{\varepsilon}D_{ij} + C_D\frac{k^3}{\varepsilon^2}\left(D_{im}D_{mi} - \frac{1}{3}D_{mn}D_{nm}\delta_{ij}\right) + C_E\frac{k^3}{\varepsilon^2}\left(\dot{D}_{ij} - \frac{1}{3}\dot{D}_{mn}\delta_{ij}\right) \quad (10)$$

where k is the turbulent kinetic energy, ε is the dissipation rate of k, and C_D and C_E are dimensionless parameters. Speziale (1987) proposed $C_D = C_E = 1.68$ for internal flows without the free-surface effect. In Eq.(10), D_{ij} and $\dot{D}_{ij}$ are the strain tensor and the tensor of the frame-independent Oldroyd derivative of D_{ij}.

At the wall and at the free surface, boundary conditions are required. It is assumed that the flow at the node closest to the wall follows the standard logarithmic law, i.e.,

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa}\ln\left(E \cdot z^+\right) \quad (11)$$

where u_* is the shear velocity and E is the logarithmic law constant. For hydraulically-smooth walls, a value of E = 9.0 is used, and for hydraulically-rough walls, the following formula by Naot and Emrani (1983) is used to estimate E:

$$E = \frac{9}{1 + \left(\frac{0.3 \cdot k_s^+}{1 + 20/k_s^+}\right)} \quad (12)$$

where k_s^+ is the non-dimensional roughness height (= $k_s u_* / \nu$, here, k_s = roughness height). The free surface is treated as a symmetric plane for all dependent variables except for the dissipation rate of the turbulent kinetic energy (ε). For ε, the relationship by Naot and Rodi (1982) is prescribed at the free surface in order to increase the dissipation level of the turbulence kinetic energy.

In the present study, the finite volume method was used to discretize the governing equations. As the solution strategy, the SIMPLER algorithm proposed by Patankar and Spalding (1972) was employed. For the discretization of the convection and diffusion terms in the transport equations, the power-law differencing scheme by Patankar (1980) was used.

Application of the 1d model

Choi and Shin (2009) applied the 1d model to the Severn River in UK (Darby and Thorne, 1996), the cross section of which is given in Figure 1. The slope of the river is 0.000194 and the width and maximum flow depth are 130 m and 8.0 m, respectively. Bed materials of the river seem to be composed of coarse gravels with D_{84} = 88 mm. Vegetation on the floodplain is 0.015 m high.

Figure 2 shows the stage-discharge curve in the Severn River, UK. Both predicted and measured data are given together with the relationship by Darby and Thorne (1996). To obtain the roughness coefficient, Darcy-Weisbach f is plotted along the width and it is converted to the roughness coefficient. Then, average values of n_m = 0.025 and n_f = 0.036 are obtained in the main channel and floodplain, respectively. It can be seen that the model predicts the stage-discharge relationship quite well. Specifically, the model over- and under-predicts the discharge for h < 4.5 m and for 4.5 m < h < 6 m, respectively. Above the bankfull level, the model slightly over-predicts the discharge, which is unsafe in flood management. The difference in the predicted results between the present and Darby and Thorne's models arises in converting the friction factor to the roughness coefficient.

Calibration of the 1d model

The non-linear k-ε model is applied to the same problem. Figure 3 shows the computed result of the streamwise mean velocity and secondary currents. It can be seen that counterclockwise-rotating and clockwise-rotating vortices, the size of which is about the flow depth, are present in the left and right hand sides of the main channel, respectively. They seem to be originated from the bottom vortices. The same trend of the secondary currents in a trapezoidal channel is reported by Knight et al. (2007). In Figure 3, the twin vortices are not seen at the juncture between the main channel and floodplain due to the relative shallow flow depth in the floodplain.

Figure 4 shows the lateral distribution of the unit discharge. Both results by the 1d model and the non-linear k-ε model are given for comparisons. Good agreement between two results is seen except for the central region of the main channel. That is, the unit discharge computed by the 1d model shows the maximum at the center of the main channel. Discharges of 365 cms and 343 cms are calculated by the 1d model and the non-linear k-ε model, respectively. If a value slightly larger than α = 0.16 is used in the 1d model, a closer agreement in the total discharge is expected.

Figure 5 shows the lateral distribution of the depth-averaged eddy viscosity. Overall shape of the profile appears to be similar to the profile of the unit discharge in Figure 4. That is, the eddy viscosity is higher in the main channel than in the floodplain. This is related with that higher turbulence kinetic energy in the main channel since Prandtl-Kolmogorov relationship was used in estimating the eddy viscosity in the RANS model.

Figure 6 shows the lateral distribution of the normalized depth-averaged eddy viscosity (α). It can be seen that α is nearly constant except for the juncture between the main channel and floodplain. This indicates that U^*h is of a similar shape of the eddy viscosity profile. The average value of α is estimated to 0.063. This is very close to the value (= 0.067) that is obtained from integrating the log-law velocity profile. In addition, Nezu and Nakagawa (1993) reported that α ranges between 0.05 – 0.06 for open-channel flows.

Discussions

In the preceding application of the 1d model, a value of the normalized eddy viscosity α = 0.16 is used. Darby (1999) proposed this value through numerical tests. That is, he changed α values in the range of 0.0 - 0.64 and obtained α = 0.16 through the best-fit between the simulated and observed data. However, the depth-averaged eddy viscosity predicted by the RANS model in Figure 5 indicates that the average value is 0.063, which is about 0.4 times the value used in the 1d model. This suggests that the normalized eddy viscosity used in the 1d model not only accounts for turbulent viscosity but also for the other effects. For example, the impact of the secondary currents, originated from the geometry of the stream, can be included.

Conclusions

In this paper, the calibration of a 1d model for the distribution of the unit discharge in a compound channel was given. The 1d model is based on the momentum balance in a cross section. The calibrated parameter is the normalized eddy viscosity that is related with turbulent shear of water columns. The RANS equations in the curvilinear coordinate system were solved with the non-linear k-ε model. The RANS model resulted in the detailed flow structure with secondary currents in the plane normal to the streamwise direction. The lateral distribution of the unit discharge by the RANS model appeared to be similar to that by the 1d model except for the center region of the main channel. The total discharge estimated by the RANS model was similar to that by the 1d model. However, the normalized eddy viscosity used in the 1d model is 2.5 times larger than that obtained by the RANS model. This is because the normalized eddy viscosity in the 1d model should account for not only the turbulent shear between water columns but also the effect of secondary currents.

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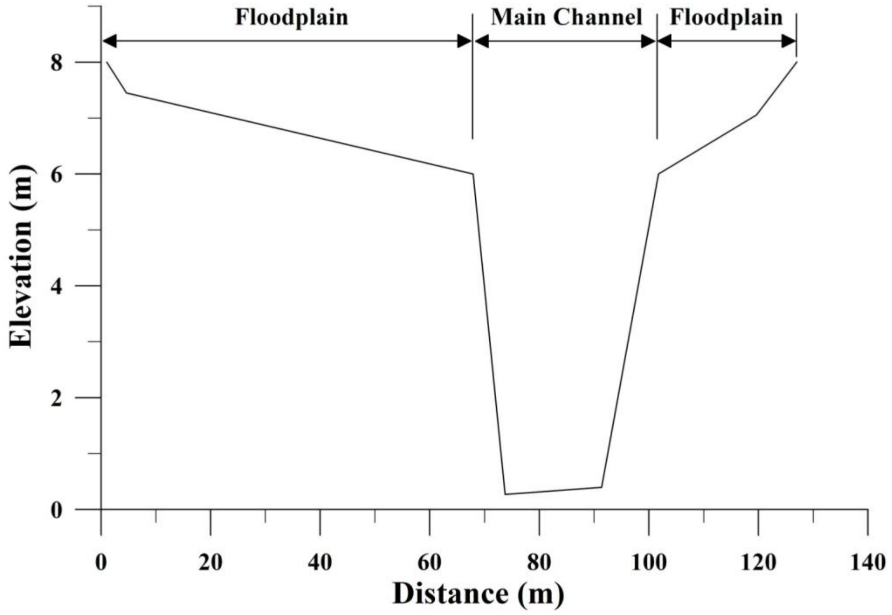

Figure 1. A cross section of the Severn River in UK

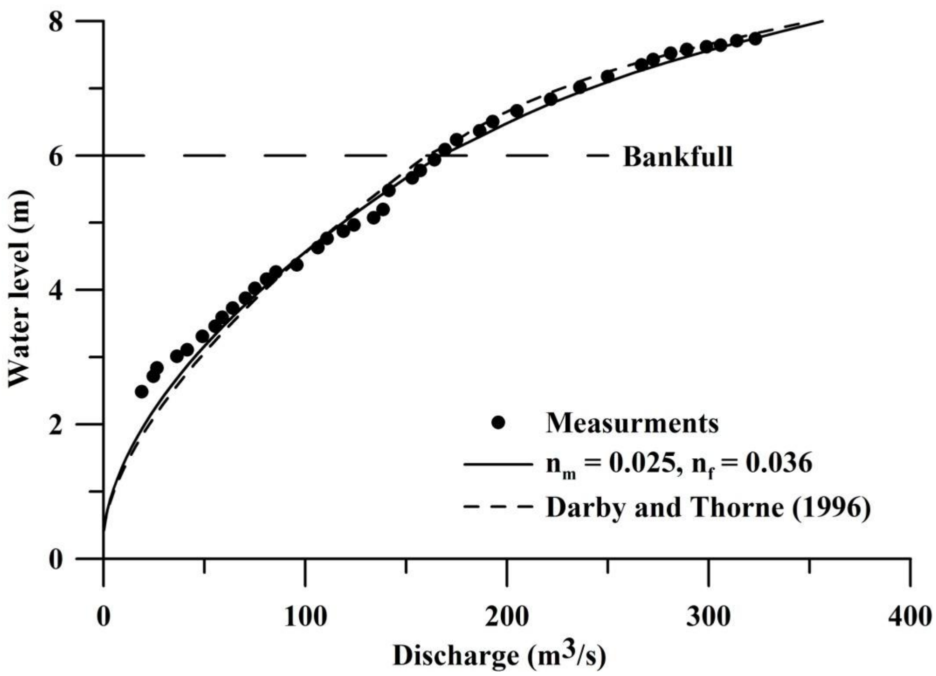

Figure 2. Stage-discharge curve in the Severn River, UK (Choi and Shin, 2009)

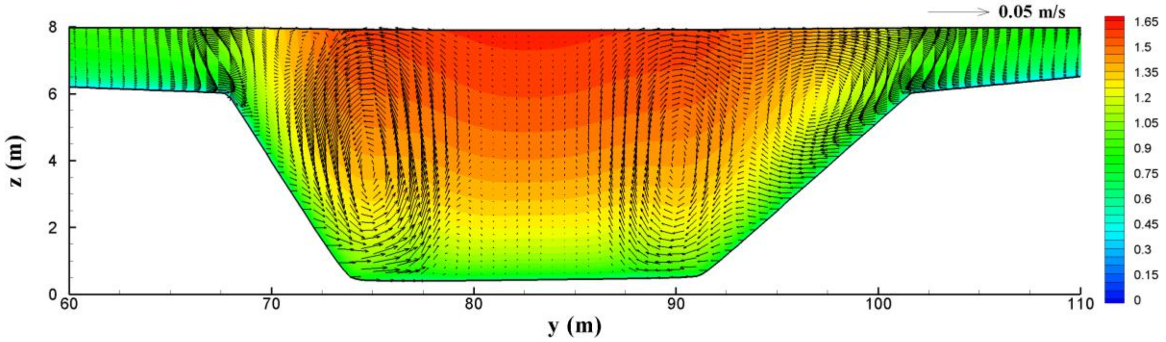

Figure 3. Streamwise mean velocity with secondary current vectors (unit, m/s)

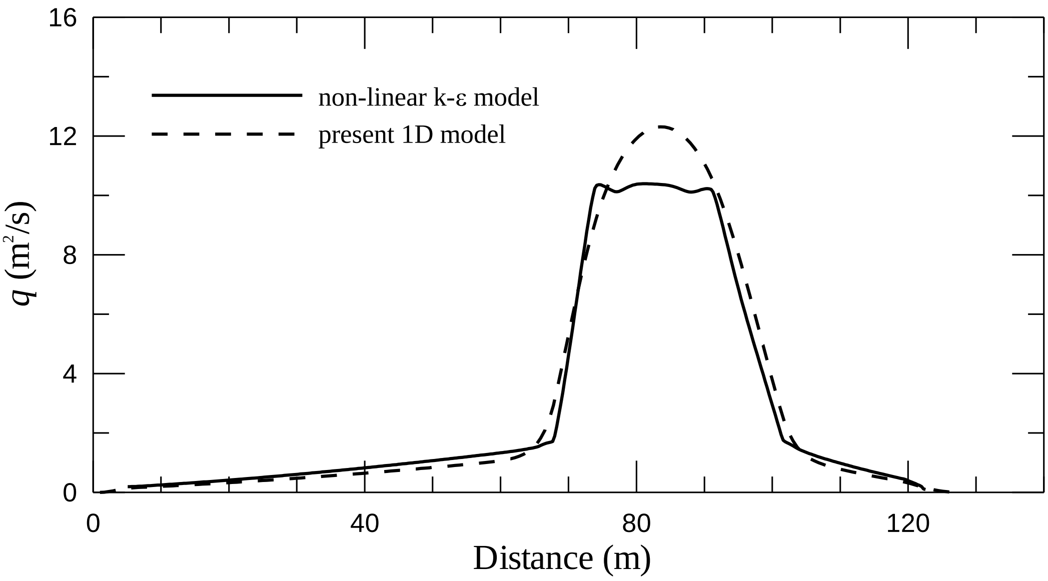

Figure 4. Lateral distribution of the unit discharge in the Severn River in UK

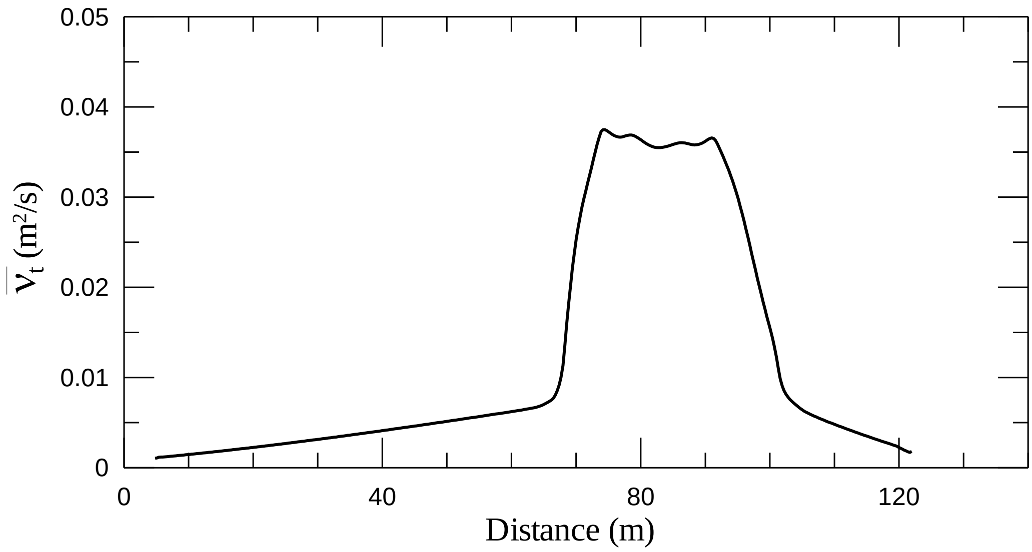

Figure 5. Lateral distribution of depth-averaged eddy viscosity ($\overline{\nu_t}$)

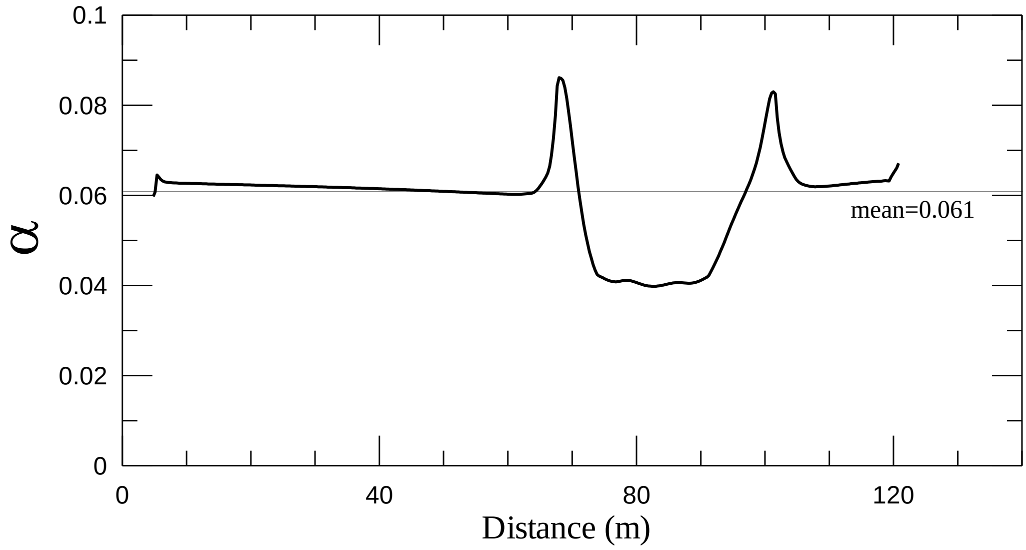

Figure 6. Lateral distribution of $\overline{\nu_t} / (U^* h)$