Numerical Model for Morphological Change of a Cross Section in Straight Channels

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1. INTRODUCTION

Due to the movement of sediment particles and mass failure, bank erosion is occurred and causes many problems such as loss of structures, inundation and expensive restoration cost. Therefore, understanding the mechanism of bank erosion is very important for predicting the change of cross section of alluvial channel including width adjustment.

With the development of computing hardware and technique over the past three decades, several studies on the bed deformation and bank erosion have been conducted. Some researchers developed numerical models to analyze temporal changes in channel forms due to bank erosion. Kikkawa et al. (1976) developed numerical model for the lateral change of cross-section in which they were considered the effects of helicoidally motion in a curved channel and the lateral bed slope. Kovacs and Parker (1994) introduced a physically-based bed load transport model which automatically deals with lateral transport. More recently, Menendez et al. (2008) proposed an integrated hydrodynamics-sedimentologic-morphologic model for the evolution of the cross section of an alluvial channel that sediment transport in the lateral direction is key issues to studying.

In this study, a numerical model capable of morphological evolution of a cross section in the straight channel is developed. The lateral distribution equation of the longitudinal flow velocity, sediment transport equation and Exner equation are numerically solved. A special algorithm to treat marginal erosion is also introduced. The numerical model is compared with simulated results of Menendez et al.'s (2008) and Kovacs and Parker's (1994) model in trapezoidal cross section of straight channel.

2. NUMERICAL MODEL

In this section, numerical model for morphological change with marginal erosion is described. It consists of hydrodynamic model, sediment transport model, morphological model, and a sliding algorithm to treat marginal erosion.

2.1 Hydrodynamic model

The governing equation for the lateral distribution of the longitudinal depth-averaged velocity is as the following which is proposed by Wark et al. (1990).

$$ghI_x - \frac{B_g f}{8}U^2 + h\frac{\partial}{\partial y}\left[\varepsilon_y \frac{\partial U}{\partial y}\right] = 0 \qquad (1)$$

where g is acceleration of gravity, h is the local flow depth, I_x (= tanα) is the longitudinal bed slope, U is the longitudinal depth-averaged flow velocity, y is the lateral coordinate, f is the Darcy-Weisbach friction factor, B_g is a

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geometrical factor ($=\sqrt{1+\tan^2\alpha+\tan^2\omega}$, where tanω is lateral bed slope), and ε_y is the lateral eddy viscosity ($=\chi_y U_* h$, where χ_y is the dimensionless lateral eddy viscosity coefficient).

In order to solve Eq. (1), numerical processes are applied as followings: At each time step, the values of Darcy-Weisbach friction factor (*f*) and dimensionless lateral eddy viscosity coefficient (χ_y) are estimated at first. Eq. (1) is discretized using a centered difference scheme. Because these discrete equations are non-linear, Newton-Raphson's method is used in this study. The boundary condition for Eq. (1) is the no-slip condition at the banks: $U|_{banks}=0$. The resultant non-linear algebraic system is used as the input value for the sediment transport model.

2.2 Sediment transport model

Suspended load and bed load are important in studying the laws of sediment movement in fluids and the processes of erosion, transportation and deposition. However, bed load is considered only in this study, because bed load is more effective on the morphological change than suspended load for the change of the cross section. Movement vector of sediment particle (or value of sediment particle velocity and angle of particle direction of motion) can be found base on the balance of three forces acting on the particle.

$$\vec{F}_D+\vec{W}_g+\vec{F}_C=0 \tag{2}$$

where $\vec{F}_D$ is the drag force due to the moving fluid which is main driving force, $\vec{W}_g$ is the component of the immersed weight of the particle tangential to the plane of the bed, and $\vec{F}_C$ is the dynamic Coulomb resistive force which represents the momentum loss due to collisions. From Eq. (2) in the orthogonal system, the algebraic equation system can be derived as following:

$$|\hat{u}_\Delta|(u_b^*-v_p^*\cos\psi)-a\tau_{C0}^*\left(|\cos\beta|\cos\psi-\frac{\sin\alpha}{\mu_c}\right)=0 \tag{3}$$

$$|\hat{u}_\Delta|v_p^*\sin\psi+a\tau_{C0}^*\left(|\cos\beta|\sin\psi-\frac{1}{\mu_c}\frac{\sin\omega.\cos^2\alpha}{\sqrt{\sin^2\omega.\cos^2\alpha+\cos^2\omega}}\right)=0 \tag{4}$$

where $|\hat{u}_\Delta|=(u_b^*+v_p^*-2.u_b^*.v_p^*\cos\psi)^{1/2}$, u_b^* is dimensionless flow velocity at the bottom, v_p^* is dimensionless particle velocity, ψ is the angle between the direction of particle motion and longitudinal flow direction, β is the angle between the vertical axis and the normal vector to the local plane of the channel bottom, a is conductance factor, and μ_c is the static friction factor. Two unknown variables, v_p^* and ψ, in the Eqs. (3) and (4) can be obtained by using Newton-Raphson method.

Bed load transport rate is calculated by the according Kovacs and Parker (1990)'s model as following:

$$\vec{q_p^*}=\zeta^*\vec{v_p^*} \tag{5}$$

$\vec{q_p^*}$ is composed of two components in the longitudinal and transversal directions. It can be expressed as following:

$$q_{bx}^*=\zeta^*.v_p^*.\cos\psi.\cos\alpha \tag{6}$$

$$q_{by}^*=\zeta^*.v_p^*.\cos\omega.(\cos\psi.\sin\alpha.\sin\omega+\sin\psi.\sqrt{\sin^2\omega.\cos^2\alpha+\cos^2\omega}) \tag{7}$$

2.3 Morphological model

Bed elevation change of a cross section is calculated by the Exner equation which is based on the sediment mass conversation condition (Raudkivi, 1990).

$$\frac{\partial z_0}{\partial t} - \frac{1}{1-\lambda}\left[\frac{\partial q_{by}}{\partial y} + \frac{\partial q_{bx}}{\partial x}\right] = 0 \tag{8}$$

where z_0 is bed elevation, t is time, and λ is the porosity of the sediment. In this study, it is assumed that the longitudinal variations in the cross section is not significant, that is, the term of $\partial/\partial x$ is equal to zero. A simple centered difference scheme is used to calculate the bed elevation change at each time step.

2.4 Sliding algorithm

After applied Exner equation, the lateral slope between successive nodes of the cross section is checked whether the transversal slope is greater than the angle of repose of bed material. If local bed slope is steeper than the angle of repose, the upper node height is lowered so that the slope angle becomes the angle of repose. And then, the corresponding node height is raised in order to accommodate the slid sediment volume.

3. VALIDATION

The numerical model is applied to experiment with the trapezoidal cross section performed by Ikeda (1981). The main parameters of experiment are shown as Figure 1. The channel is straight, prismatic and symmetric, with a longitudinal slope of 0.00215, width of 0.220 m and 1V/1.8H bank slope. The initial depth is 0.061 m. Both the channel bottom and banks are made of sand with mean diameter d = 1.3 mm. The parameter values adopted for the developed model are $a^{1/2}$ = 11.9, $\tau^*_{c0} = 0.035$, μ_c = 0.84, λ=0.35, and χ = 0.13. The initial lateral domain is divided into 100 intervals with Δy = 0.0044 m. The time step is set up as Δt = 1 second for the stability of calculation. The results are shown in Figures 2 and 3. When the results are compared with that of Menendez et al. (2008) and Kovacs and Parkers (1994), the resultant trend is similar. Deposition and erosion are occurred in around of main channel and margin, respectively. However, deposition rate around main channel is underestimated compared with other's results.

4. CONCLUSION

This paper presented a numerical model that is capable of predicting morphological change with marginal erosion of a cross section in straight channels. Hydrodynamic model, sediment transport model, morphological model, and a sliding algorithm to treat marginal erosion were adopted and solved numerically. The developed model was applied to the experiment performed by Ikeda (1981) in trapezoidal cross section. It was shown that the results were very similar with results of Menendez et al. (2008)'s and Kovacs and Parkers (1994)'s model. The model can properly simulate the erosion at the margin and the deposition around main channel.

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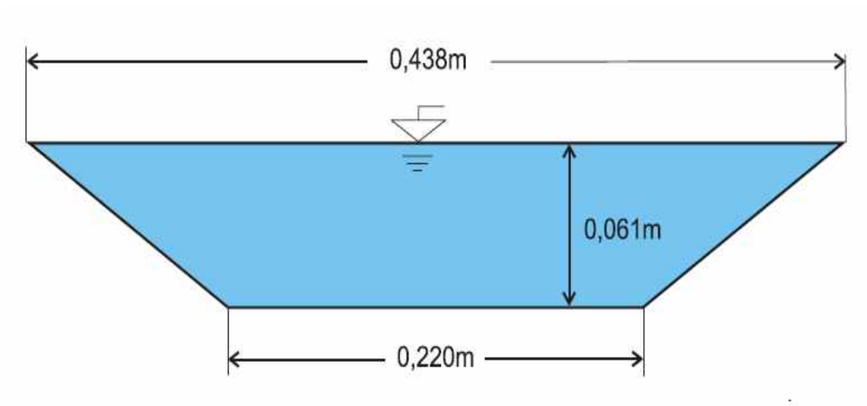

Figure 1. Initial form of cross section (Ikeda, 1981)

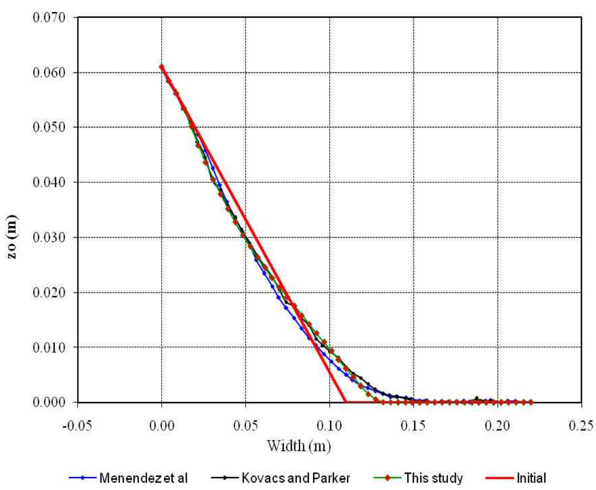

Figure 2. Change of cross section at 60 seconds.

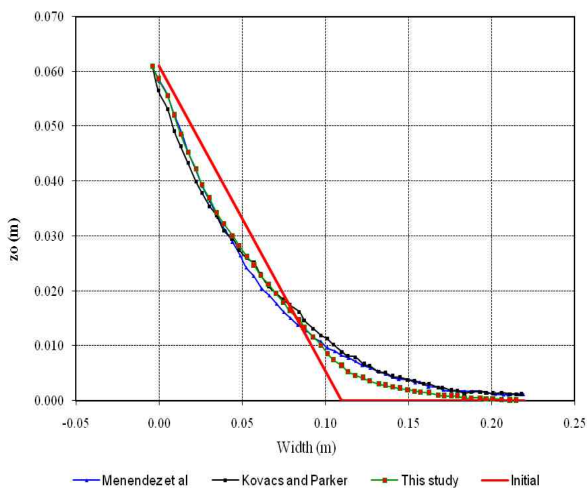

Figure 3. Change of cross section at 404 seconds.